$B \to \rho\pi$ and the unitary angle α

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Abstract

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The role of the decay mode $B \to \rho\pi$ for the determination of the unitarity angle α is critically examined in view of the smaller than expected ratio $\mathcal{B}(B \to \rho^\pm\pi^\mp)/\mathcal{B}(B \to \rho^0\pi^\pm)$ found by the CLEO collaboration.

1 Introduction

The task of determining the angle α is complicated by the problem of separating two different weak hadronic matrix elements, each carrying its own weak phase. The evaluation of these contributions, referred to in the literature as the *tree* (T) and the *penguin* (P) contributions, suffers from the common theoretical uncertainties related to the estimate of composite four-quark operators between hadronic states. For these estimates, only approximate schemes, such as the factorisation approximation, exist at the moment, and for this reason several ingenuous schemes have been devised, trying to disentangle T and P contributions. In general one tries to exploit the fact that in the P amplitudes only the isospin–1/2 [a] part of the non–leptonic Hamiltonian is active in the decay $B \to \pi\pi$ [1]; by measurements involving several different isospin amplitudes, one can separate the two amplitudes and get rid of the ambiguities arising from the ill–known penguin matrix elements. However such measurements are not simple, for example $B \to \pi^0\pi^0$ is probably small and difficult to detect; the recent measurement by CLEO [2] of the $B \to \pi^+\pi^-$ branching ratio shows that penguin contributions are certainly not small; discrete ambiguities reduce the predictive power of the isospin method.

In order to have an alternative measurement of the angle α different strategies were proposed, either involving all the decay modes of a B into a $\rho\pi$ pair as well as three time–asymmetric quantities measurable in the three channels for neutral B decays, or attempting to measure only the neutral B decay modes by looking at the time-dependent asymmetries in different regions of the Dalitz plot [3,4].

Preliminary to these analyses is the assumption that, using cuts in the three invariant masses for the pion pairs, one can extract the ρ contribution without significant background contaminations. The ρ has spin 1, the π spin 0 as well as the initial B, and therefore the ρ has angular distribution $\cos^2\theta$ (θ is the angle of one of the ρ decay products with the other π in the ρ rest frame). This means that the Dalitz plot is mainly populated at the border, especially the corners, by this decay. Only very few

[a] If one neglects electroweak penguins.

events should be lost by excluding the interior of the Dalitz plot, which is considered a good way to exclude or at least reduce backgrounds. Analyses following these hypotheses were performed by the BaBar working groups [5]; MonteCarlo simulations, including the background from the f_0 resonance, show that, with cuts at $m_{\pi\pi} = m_\rho \pm 300$ MeV, no significant contributions from other sources are obtained. Also the role of excited resonances such as the ρ' and the non–resonant background has been discussed [6].

A signal of possible difficulties for this strategy arises from new results from the CLEO Collaboration [7]:

$$\mathcal{B}(B^\pm \to \rho^0\pi^\pm) = (10.4^{+3.3}_{-3.4} \pm 2.1) \times 10^{-6} , \tag{1}$$

$$\mathcal{B}(B \to \rho^\mp\pi^\pm) = (27.6^{+8.4}_{-7.4} \pm 4.2) \times 10^{-6} , \tag{2}$$

with a ratio

$$R = \frac{\mathcal{B}(B \to \rho^\mp\pi^\pm)}{\mathcal{B}(B^\pm \to \rho^0\pi^\pm)} = 2.65 \pm 1.9 . \tag{3}$$

As discussed in [7], this ratio looks rather small; as a matter of fact, when computed in simple approximation schemes, as factorisation with no penguins, one gets, from the WBS model [8], $R \simeq 6$ (using $a_1 = 1.02$, $a_2 = 0.14$). The inclusion of penguin contributions in the factorisation approximation does not help explaining the experimental result as one gets $R \simeq 5.5$. One may wonder if the factorisation approximation is too rough to give an accurate result or if penguin contributions are larger than expected, in order to explain the experimental result. An estimate of non–factorisable contributions was given by Martinelli and co–workers [9] by parameterising these contributions in terms of the known $B \to \pi K$ decays. Using their estimates we deduced values for the ratio R (see Table 1) and they all agree with the result obtained in the factorisation approximation. Therefore some other reason may be advocated for explaining the experimental result for the ratio R.

Table 1: Estimates for the ratio R beyond the factorisation approximation (the so–called charming penguins) using different sets of input data: QCD sum–rules (QCDSR), lattice–QCD, quark models (QM)

QCDSR	$R = 6.3$
lattice	$R = 5.5$
QM	$R = 6.4$

In [10] we showed that a new contribution, not discussed before, is indeed relevant to the decay of a charged B to $\rho\pi$ and to a lesser extent to the decay of a neutral B to $\rho\pi$. It arises from the virtual resonant production depicted in Fig. 1, where the intermediate particle is the B^* meson resonance or other excited states. The B^* resonance, because of phase–space limitations, cannot be produced on

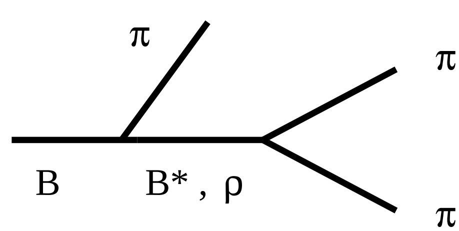

Fig. 1 - The polar diagram. For the B resonances ($B^* = 1^-$, 0^+) the strong coupling is on the left and the weak coupling on the right; the situation is reversed for the ρ production.

the mass shell. Nonetheless the B^* contribution might be important, owing to its almost degeneracy in mass with the B meson; therefore its tail may produce sizeable effects in some of the decays of B into light particles, also because it is known theoretically that the strong coupling constant between B, B^* and a pion is large [11]. Concerning other states, we expect their role to decrease with their mass, since there is no enhancement from the virtual particle propagator; we shall only consider the 0^+ state B_0 with $J^P = 0^+$ because its coupling to a pion and the meson B is known theoretically to be uniformly (in momenta) large [11].

2 Interaction Hamiltonian

The effective weak non-leptonic Hamiltonian for the $|\Delta B| = 1$ transition is[b]:

$$H = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{ud} \sum_{k=1}^{2} C_k(\mu) Q_k - V_{tb}^* V_{td} \sum_{k=3}^{6} C_k(\mu) Q_k \right\} . \tag{4}$$

We use the following values of the Wilson coefficients: $C_1 = -0.226$, $C_2 = 1.100$, $C_3 = 0.012$, $C_4 = -0.029$, $C_5 = 0.009$, $C_6 = -0.033$; they are obtained in the HV scheme [12], with $\Lambda_{\bar{MS}}^{(5)} = 225$ MeV, $\mu = \bar{m}_b(m_b) = 4.40$ GeV and $m_t = 170$ GeV. For the CKM mixing matrix we use the Wolfenstein parameterisation with $\rho = 0.05$, $\eta = 0.36$ and $A = 0.806$ in the approximation accurate to order λ^3 in the real part and λ^5 in the imaginary part, i.e. $V_{ud} = 1 - \lambda^2/2$, $V_{ub} = A\lambda^3\ [\rho - i\eta\ (1 - \lambda^2/2)]$, $V_{td} = A\lambda^3(1 - \rho - i\eta)$ and $V_{tb} = 1$.

The diagram of Fig. 1 describes two processes. For the B^* intermediate state there is an emission of a pion by strong interactions, followed by the weak decay of the virtual B^* into two pions; for the ρ intermediate state there is a weak decay of $B \to \rho\pi$ followed by the strong decay of the ρ resonance. We compute these diagrams as Feynman graphs of an effective theory within the factorisation approximation, using information from the effective Lagrangian for heavy and light mesons [13] and form factors for the couplings to the weak currents.

3 $B \to \rho\pi$ Decays

For the charged B decays we obtain the results in Table 2, with $g = 0.40$ and $h = -0.54$, which lie in the middle of the allowed ranges for these parameters. The branching ratios are obtained with $\tau_B = 1.6$ psec and, by integration over a limited section of the Dalitz plot, defined as $m_\rho - \delta \leq (\sqrt{t}, \sqrt{t'}) \leq m_\rho + \delta$ for $B^- \to \pi^-\pi^-\pi^+$ and $m_\rho - \delta \leq (\sqrt{s}, \sqrt{s'}) \leq m_\rho + \delta$ for $B^- \to \pi^-\pi^0\pi^0$. For δ we take 300 MeV. This amounts to require that two of the three pions (those corresponding to the charge of the ρ) reconstruct the ρ mass within an interval of 2δ.

Table 2: Effective branching ratios for the charged B decay channels into three pions for the choice of the strong coupling constants $g = 0.40$ and $h = -0.54$. Cuts as indicated in the text.

Channels	ρ	$\rho + B^*$	$\rho + B^* + B_0$
$B^- \to \pi^-\pi^0\pi^0$	1.0×10^{-5}	1.0×10^{-5}	1.0×10^{-5}
$B^- \to \pi^+\pi^-\pi^-$	0.41×10^{-5}	0.58×10^{-5}	0.63×10^{-5}

We can notice that the inclusion of the new diagrams (B resonances in Fig. 1) produces practically no effect for the $B^- \to \pi^-\pi^0\pi^0$ decay mode, while for $B^- \to \pi^+\pi^-\pi^-$ the effect is significant. In Table 2 the overall effect is an increase of 50% of the branching ratio as compared to the result obtained by the ρ resonance alone. It should be observed that the events arising from the B resonances diagrams represent an irreducible background. The contributions from the B resonances populate the whole Dalitz plot and, obviously, cutting around $t \sim t' \sim m_\rho$ significantly reduces them. Nevertheless their effect can survive the experimental cuts, since there can be enough data at the corners, where the contribution from the ρ dominates. Integrating on the whole Dalitz plot, with no cuts and including all contributions, gives $\mathrm{B}r(B^- \to \pi^-\pi^0\pi^0) = 1.5 \times 10^{-5}$ and $\mathrm{B}r(B^- \to \pi^+\pi^-\pi^-) = 1.4 \times 10^{-5}$ where the values of the coupling constants are as in Table 2.

We now turn to the neutral B decay modes. The results in Table 3 show basically no effect for the $\bar{B}^0 \to \rho^\pm\pi^\mp$ decay channels and a moderate effect for the $\rho^0\pi^0$ decay channel. The effect in this channel is of the order of 20% (resp. 50%) for $\bar{B}^0$ (resp. B^0) decay, for the choice $g = 0.60$, $h = -0.70$ (the one maximising the effect of the B^* resonances); for smaller values of the strong coupling constants the effect is reduced.

[b] We omit, as usual in these analyses, the electroweak operators Q_k (k = 7, 8, 9, 10); they are in general small, but for Q_9, whose role might be sizeable; its inclusion in the present calculations would be straightforward.

Table 3: Effective branching ratios for the neutral B decay channels into $\rho\pi$ ($g = 0.60$, $h = -0.70$). Cuts as indicated in the text.

Channels	ρ	$\rho + B^*$	$\rho + B^* + B_0$
$\bar{B}^0 \to \rho^-\pi^+$	0.50×10^{-5}	0.52×10^{-5}	0.49×10^{-5}
$\bar{B}^0 \to \rho^+\pi^-$	1.7×10^{-5}	1.7×10^{-5}	1.7×10^{-5}
$\bar{B}^0 \to \rho^0\pi^0$	0.10×10^{-5}	0.15×10^{-5}	0.12×10^{-5}
$B^0 \to \rho^+\pi^-$	0.49×10^{-5}	0.51×10^{-5}	0.48×10^{-5}
$B^0 \to \rho^-\pi^+$	1.7×10^{-5}	1.7×10^{-5}	1.7×10^{-5}
$B^0 \to \rho^0\pi^0$	0.11×10^{-5}	0.17×10^{-5}	0.15×10^{-5}

Integration on the whole Dalitz plot, including all contributions, gives $\mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-\pi^0) = 2.6 \times 10^{-5}$ confirming again that most of the branching ratio is due to the ρ-exchange (the first three lines of the ρ column in Table 3 sum up to 2.3×10^{-5}).

4 Conclusions

The effect of including B resonance polar diagrams is significant for the $B^\mp \to \pi^\mp\pi^\mp\pi^\pm$ decay and negligible for the other charged B decay mode. This result is of some help in explaining the recent results from the CLEO Collaboration, since we obtain $R = 3.5 \pm 0.8$, to be compared with the experimental result in eq. (3). The ρ resonance alone would produce a result up to a factor of 2 higher. Therefore we conclude that the polar diagrams examined [10] are certainly relevant in the study of the charged B decay into three pions. Concerning the neutral B decays which are relevant for the determination of the unitarity angle α, only the $\rho^0\pi^0$ decay channel is partially affected by the extra contributions we considered.

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